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# Heat and momentum transfer in fluids heated in tubes with turbulence generators at moderate Prandtl and Reynolds numbers

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**Abstract**

Experimental investigations were performed by heating streams of air, water and glycerine in tubes of different diameters furnished with perforated baffles of various types. The results are presented as a correlation between Nusselt and Prandtl numbers and modified Reynolds number related to the drag coefficient specific for baffle type. Concerning momentum transfer and fluid flow field in the system, a method has been elaborated to predict the drag coefficient in relation to the baffle parameters. This enables us to predict the heat transfer rate and the flow resistance by scaling up the system. 

**Nomenclature**

$A$ factor in equations (3), (17), (18) and (32)
$A(T)$ factor in equations (27)–(29)
$B$ factor in equations (21) and (22)
$c_p$ heat capacity of fluid at constant pressure [kJ kg$^{-1}$ K$^{-1}$]
$d$ diameter of opening in baffle [m]
$d_e$ effective linear dimension of baffle [m]
$d_h$ hydraulic diameter of opening in baffle [m]
$D$ inside diameter of heat exchanger tube [m]
$D_0$ outside diameter of heat exchanger tube [m]
$G$ mass flow rate of fluid [kg s$^{-1}$]
$l$ spacing of baffles in set [m]
$L$ length of heat exchanger tube [m]
$L_c$ length of central section of heat exchanger tube [m]
$L_e$ length of heated section of heat exchanger tube [m]
$n$ number of baffles
$n_0$ number of openings in baffle
$Nu$ Nusselt number, $\alpha D/\lambda$
$p$ static pressure [Pa]
$Pr$ Prandtl number, $c_p\mu/\lambda$
$q_i$ local heat flux [W]
$Q_{el}$ electrical power input [W]
$Re$ Reynolds number, $uD\rho/\mu$
$Re^+$ modified Reynolds number, $Re(\xi/8)^{1/2}$
$t_i$ local temperature of wall of heat exchanger tube [°C]
$t_i^*$ local temperature of fluid [°C]
$t_s$ temperature of surrounding air [°C]
$t'$ local temperature of outer surface of heat exchanger insulation [°C]
$u$ linear velocity of fluid, averaged over cross-section of heat exchanger tube [m s$^{-1}$]
$\hat{u}_a$ velocity of fluid, averaged over opening cross-section [m s$^{-1}$]
$\hat{u}_m$ maximal linear fluid velocity at opening centre [m s$^{-1}$]
$\hat{u}_s$ local linear velocity within stagnation zone [m s$^{-1}$]
$u^+$ shear stress velocity of fluid [m s$^{-1}$]
$V$ volumetric flow rate of fluid [m$^3$ s$^{-1}$]
$x$ axial coordinate [m].

*Greek symbols*
$\alpha$ heat transfer coefficient [W m$^{-2}$ K$^{-1}$]
$\varepsilon$ perforation of baffle
$\lambda$ coefficient of molecular thermal conductivity of fluid [W m$^{-1}$ K$^{-1}$]
$\lambda_{Cu}$ coefficient of molecular thermal conductivity of copper [W m$^{-1}$ K$^{-1}$]
$\mu$ coefficient of dynamic viscosity of fluid [kg m$^{-1}$ s$^{-1}$]
$\xi$ drag coefficient of baffle set in equations (2), (4) and (6)

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$\xi_i$ drag coefficient of isolated baffle in equations (7) and (8)
$\xi^+$ drag coefficient of single baffle in set in equations (5) and (6)
$\rho$ density of fluid [kg m$^{-3}$].

## 1. Introduction

This work is an extension of our formerly initiated experimental investigation concerning augmentation of heat transfer between the fluid stream and internal surface of a tube wall [1, 2] consisting of furnishing the tube with inserts of perforated disc-baffles distributed perpendicularly to its axis.

In our previous papers [3, 4] on the subject we discussed other commonly accepted passive methods for augmenting heat transfer within tubes. Especially, we analysed available information concerning augmentation effects within tubes with internally roughened or ribbed walls and in tubes furnished with inserts of twisted tapes or with wire-net matrices in comparison with results of our experiments with heating air stream within tubes furnished with our inserts with various geometrical parameters. We also presented two criteria for comparative estimation of the effectiveness of various means proposed for heat transfer augmentation within tubes. One of these criteria is based on comparison of heat transfer surface areas in an exchanger with tubes furnished with augmenting means and in a conventional exchanger when, respectively, the same amounts of energy, mass and momentum are exchanged (the same exploitation costs are assumed in both), but with different numbers of tubes (lengths and diameters) as well as fluid velocities within the tubes. The other criterion is based on analogous comparison of the heat transfer surface areas regardless, however, of limitation in momentum amount exchanged in the exchanger with tubes furnished with augmenting means. This criterion is convenient when the investment cost of the designed exchanger is high but high pressure drop in the fluid streaming within the tubes of the apparatus is acceptable. Thus, considerable reduction in heat transfer surface area is possible.

On the basis of these criteria, employing data taken from the literature and our experimental results, we compared the effectiveness of a tube furnished with our perforated disc-baffles, with twisted tapes and with internally ribbed walls [5, 6]. The results of experiments and of computations indicate that insertion of a set of perforated disc-baffles into a tube enables us to increase the heat transfer rate between the tube wall and fluid streaming within the tube up to seven times as compared with that observed at the same fluid flow rate in an ordinary tube with the same diameter. The other compared augmenting means increase the heat transfer rate up to three times only.

The comparison of the reductions in heat transfer surface area, estimated at the same amounts of energy, mass and momentum exchanged in the exchangers, each with tubes furnished with one of the above mentioned three augmenting means indicates approximately the same effects when narrow tubes are used in the compared exchangers. However, when tubes with larger diameters are considered, inserts of perforated baffles are evidently most effective. They also demonstrate an advantage essential for scaling-up the heat exchangers with augmented heat transfer within tubes under dynamic similarity condition. Namely, the exponent at Reynolds number characterizing dynamic similarity of heat transfer augmentation is constant within rather a broad range of Reynolds numbers. It has been stated [3, 4] that the conventionally defined Reynolds number cannot be employed as the dynamic similarity criterion in scaling up of heat exchangers with tubes furnished with sets of perforated disc-baffles differing in their geometrical parameters. It has been found, however, that the dynamic similarity of the heat transfer in the proposed systems can be conveniently described by the modified Reynolds number

$$Re^+ = u^+ D\rho/\mu \tag{1}$$

based on the shear stress fluid velocity defined by the formula

$$u^+ = u(\xi/8)^{1/2} \tag{2}$$

in which $\xi$ denotes an over-all drag coefficient specific for a given baffle set. Thus, it has been established that heat transfer between the hot wall of a tube and a fluid flowing within it can be modelled by means of the correlation

$$Nu = A\, Pr^m (Re^+)^k. \tag{3}$$

The dimensionless factor, $A$, dependent on the baffle type and the exponent, $k$, for $Re^+$ were determined experimentally ($0.7 \leqslant k \leqslant 0.8$), but the exponent $m = 0.4$ was assumed to be the same as for heating fluids in common tubes.

Equation (3) could be employed in predicting heat transfer rate in a tube furnished with a given baffle set if $\xi$ was known. This, as yet, could only be determined in relation to the pressure drop, $\Delta p$, measured in the fluid stream for a given baffle set, as it follows from the relation

$$\xi = 2\Delta p D/\rho u^2 L. \tag{4}$$

The over-all drag coefficient, $\xi$, can be replaced by the partial drag coefficient, $\xi^+$, specific for a single baffle in a set, as defined by

$$\xi^+ = 2\Delta p/n\rho u^2. \tag{5}$$

Both coefficients, $\xi$ and $\xi^+$, are related to each other by the formula

$$\xi = \xi^+ nD/L = \xi^+(L/l)(D/L) = \xi^+(d/l)(D/d). \quad (6)$$

It has to be noted that the value of $\xi^+$ differs from that of $\xi_i$, specific for a single isolated baffle, and defined as

$$\xi_i = 2\Delta p_i/\rho u^2 \quad (7)$$

which can be evaluated by means of the correlation

$$\xi_i = \{[1+0.707\sqrt{(1-\varepsilon)}-\varepsilon]/\varepsilon\}^2 \quad (8)$$

recommended in literature [7] for baffles with openings of arbitrary shape and linear dimension when their free surface area, $\varepsilon$, falls into the range $0.01 \leqslant \varepsilon \leqslant 0.9$.

In order to rid equation (3) of empirical factors, this work has been aimed at finding relations between $\xi^+$ and $\xi_i$, as well as between factor $A$ and the parameters of the baffle. This will facilitate the predicting of the heat transfer augmentation and of the flow resistance in the proposed system.

This work has also been aimed at verification of the contribution of the conventionally defined Prandtl number in equation (3). This problem is especially important from the point of view of modelling heat transfer augmentation in liquids or compressed gases heated within tubes of a tube-in-shell exchanger.

This work has consisted of:

- experimental determination of $Nu$ and of $\xi^+$ during heating streams of various fluids in tubes differing in diameters and furnished with sets of baffles differing in spacing and in geometry;
- experiments and theoretical considerations concerning fluid flow patterns in these tubes in order to find the relation between $\xi^+$ and the baffle parameters;
- correlating $Nu$ with $\xi^+$ and thus with $Re^+$.

## 2. Experiments

Several series of experiments were performed with the baffle types illustrated in Fig. 1. The baffles were made of steel sheet, 0.5 mm thick, and they were assembled perpendicularly to the tube axis on a coaxial rod in sets differing in spacing along the tube in individual series of experiments.

Experiments were performed at steady-state with air, water and aqueous solutions of glycerine, 40% and 58%. The fluid velocities, related to the tube cross-section, were varied in individual groups of experiments in the ranges: for air $4 \leqslant u \leqslant 20$ m s$^{-1}$, and for liquids $4\times10^{-2} \leqslant u \leqslant 6\times10^{-1}$ m s$^{-1}$ and they corresponded with the conventional $Re$ in the range $7\times10^2 \leqslant Re \leqslant 3\times10^4$.

The tests with heating air streams were performed with the tubes 35.5, 70 and 100 mm in diameter, used as heat exchangers with constant wall temperatures. For comparison, however, several series of tests with heating air streams were performed in a tube 30 mm in diameter, used as a heat exchanger with constant heat flux on its wall in all experiments with liquids.

Figure 2 illustrates schemes of experimental stands. An open system, presented in Fig. 2(a), was employed in experiments performed with air. Ambient air was pumped through a heat exchanger 1 by a blower 2, and its flow rate was controlled by a rotameter 3. The air stream was heated in a heat exchanger of the tube-in tube type with the inner tube heated by steam saturated at atmospheric pressure. Thus, the air stream was heated at the constant temperature of the tube wall. Details are given elsewhere [3]. A closed system, shown in Fig. 2(b), was employed in experiments performed with liquids. Cold liquid was sucked from a reservoir 4, and pumped by a pump 5 through another reservoir 6 into a heat exchanger 1. Reservoir 6 was equipped with an electric heater controller by a regulator maintaining the temperature of the liquid stream at the heat exchanger inlet at constant level $t_0 = 30$°C during each experiment. The liquid flow rate was regulated by a set of valves 7 and controlled by a rotameter 3. Hot liquid, leaving the heat exchanger 1, was turned back through a cooler 8 into reservoir 4. A scheme of the heat exchanger with constant heat flux on its wall, used interchangeable in both stands, is illustrated in Fig. 3. A copper tube 1, with inside diameter $D = 30$ mm and outside diameter $D_0 = 40$ mm with length $L = 700$ mm, was heated from outside by an electric heater at a constant heat flux $Q_{el}/L_e$. The tube was isolated from outside in order to reduce heat loss. Figure 3 also presents the tube wall element 2. Resistance wire 3 was coiled uniformly in a groove made in the tube wall and it was electrically isolated by a paste 4 with high electrical resistance and with high thermal conductivity. The distribution of the local temperatures, $t_i$, along the tube wall was measured by a set of thermocouples 5 uniformly spaced at a distance, $\Delta x_i$, and fixed in cavities filled with a paste 6 with high thermal conductivity. Both ends of tube 1 were isolated thermally from inlet and outlet metal tubings by means of nylon connectors 7 in order to eliminate heat loss in a longitudinal direction. The fluid stream temperatures at the initial and at the final cross-section of the heat exchanger were measured by means of two sets of thermocouples 8. Each set was made of four units with junctions located at radial positions, chosen so that the average of their indications was equal to the mean temperature of the fluid in each cross-section of the apparatus. The pressure drop in the fluid stream was measured by a manometer connected with taps 9 at the ends of the heat exchanger.

The pressure drops needed for evaluation of $\xi^+$, and the data needed for evaluation of $Nu$, were determined simultaneously in individual experiments.

The method for collecting data necessary for evaluation of $Nu$ for the tubes with constant wall temperature, and the evaluation itself, were described previously [3].

The $Nu$-values for the tube with constant heat flux on

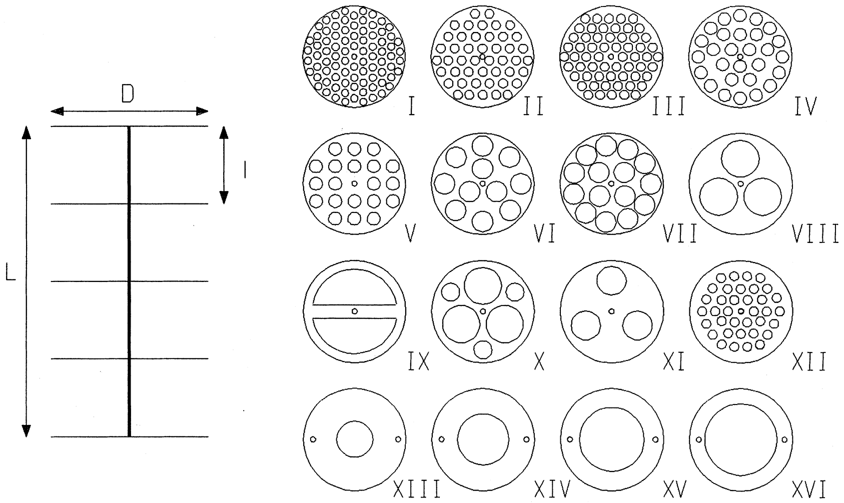



Fig. 1. Investigated baffles and their characteristics $D$ = 30, 33.5, 70 and 100 mm.

| T | $d/D$ | $\varepsilon$ | $n_0$ |
|---|---|---|---|
| I | 0.0714 | 0.44 | 85 |
| II | 0.100 | 0.47 | 46 |
| III | 0.100 | 0.61 | 60 |
| IV | 0.121 | 0.42 | 28 |
| V | 0.143 | 0.42 | 20 |
| VI | 0.200 | 0.45 | 11 |
| VII | 0.200 | 0.61 | 15 |
| VIII | 0.400 | 0.49 | 3 |
| IX | 0.549* | 0.61 | 2 |
| X | 0.334* | 0.61 | 6 |
| XI | 0.320 | 0.30 | 3 |
| XII | 0.961 | 0.40 | 36 |
| XIII | 0.333 | 0.167 | 1 |
| XIV | 0.510 | 0.318 | 1 |
| XV | 0.633 | 0.461 | 1 |
| XVI | 0.700 | 0.555 | 1 |

* hydr. diam.

its wall were evaluated as the averages of the sets of local values calculated on the ground of the data collected experimentally along the tube. The calculations of the local values, $Nu_i$, were performed according to the formula:

$$Nu_i = \frac{\alpha_i D}{\lambda} = \frac{q_i}{\pi\lambda\left(\frac{\Delta x_i}{2} + \frac{\Delta x_{i-1}}{2}\right)\Delta t_i^*}. \tag{9}$$

The local difference of the wall and of the fluid temperatures, $\Delta t_i^*$ necessary for evaluation of $Nu_i$ was estimated according to the scheme presented in Fig. 4. The local average temperature of the fluid in the initial cross-section of the exchanger was measured experimentally by the thermocouples 8 shown in Fig. 3. As the fluid mass flow rate, $G$, and the local heat flux, $q_i$, transferred to the fluid stream through the tube segment, $\Delta x_i$, were known during each experiment, the local fluid temperature, $\Delta t_i^*$, average in the succeeding tube cross-section, corresponding to the longitudinal position of the thermocouple measuring the tube wall temperature, was evaluated according to the balance equation

$$t_i^* = t_{i-1}^* + q_i/Gc_{pi}. \tag{10}$$

The local heat flux, $q_i$, was evaluated according to the formula

$$q_i = (q_{el,i} - q_{s,i})\left(\frac{\Delta x_{i-1} + \Delta x_i}{2}\right) + q_{x,i}. \tag{11}$$

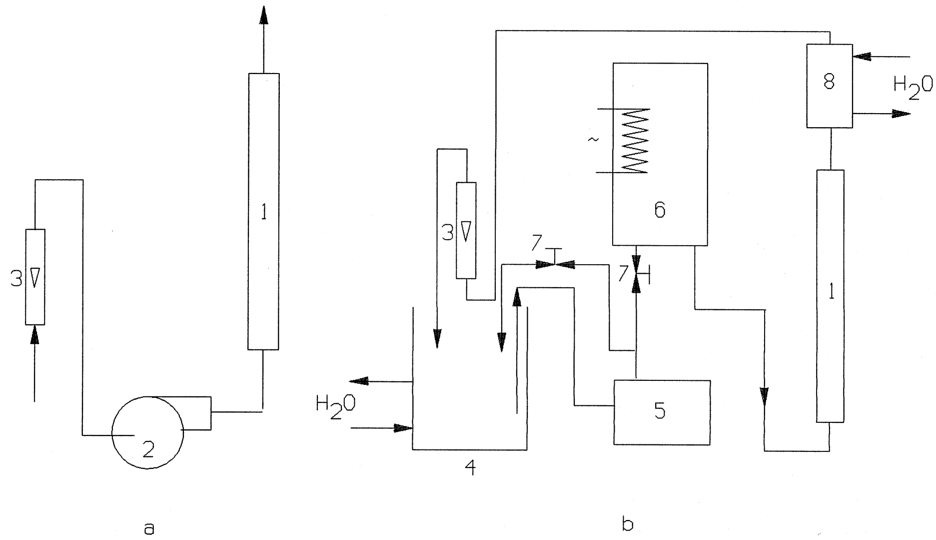



Fig. 2. Experimental stands (a) for air, (b) for liquids.

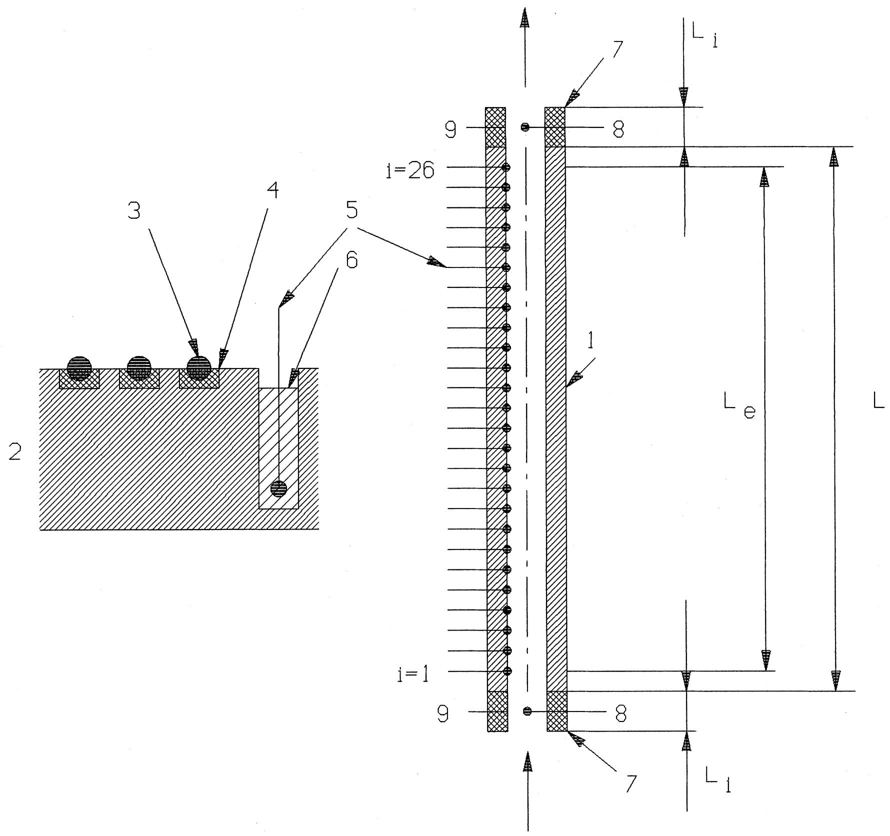



Fig. 3. Heat exchanger.

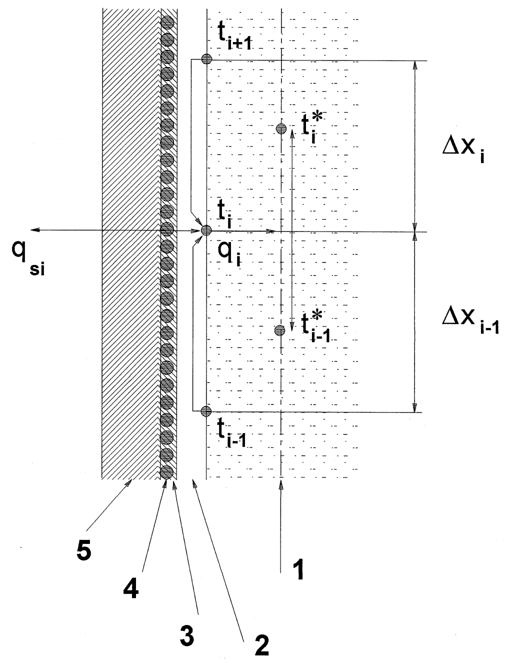



Fig. 4. Explanation for evaluation of heat balance in system: heat exchanger wall—element of fluid stream.

The heat flux, $q_{el,i}$, generated by the electrical resistance coil, was evaluated by means of the formula

$$q_{el,i} = Q_{el}/L_e. \tag{12}$$

The heat flux, $q_{x,i}$, transferred into the $i$th segment of the tube wall due to thermal conductivity of its material, was estimated according to the equation:

$$q_{x,i} = \frac{\pi}{4}(D_0^2 - D^2)\frac{\lambda_{Cu}}{\Delta x_i}(t_{i+1} - t_{i-1}). \tag{13}$$

The heat loss from the outer surface of the $i$th segment of the exchanger, $q_{s,i}$, was estimated after the relation:

$$q_{s,i} = \alpha_{s,i}(t_i' - t_s)\left(\frac{\Delta x_{i-1} + \Delta x_i}{2}\right). \tag{14}$$

The coefficient, $\alpha_{s,i}$ was evaluated after the empirical correlation proposed by Hobler [8]. The heat loss through the nylon tubings at both ends of the exchanger did not exceed 2% in the experiments with air, and 0.5% in the tests with liquids. The local arithmetical average of the differences of the wall and of the fluid temperature at the initial and at the final cross-section of the $i$th segment of the exchanger was calculated as:

$$\Delta t_i^* = t_i - \frac{t_i^* + t_{i-1}^*}{2}. \tag{15}$$

The Prandtl number was evaluated on the basis of the fluid molecular properties related to the arithmetical average of the fluid temperatures at the inlet and at the outlet of the heat exchanger; its values have been changed in the range $0.7 \leqslant Pr \leqslant 34$.

## 3. Results

In order to validate the accuracy of the experimental facilities, several sets of experiments with heating gas and liquids streaming within an ordinary tube were performed. The results were compared with the appropriate values calculated according to the commonly accepted correlation

$$Nu/Pr^{0.4} = 0.023 \cdot Re^{0.8}. \tag{16}$$

Figure 5 illustrates excellent agreement between experimental and calculated data.

Figure 6 shows examples of the longitudinal profiles of the exchanger wall temperature, $t_i = f(x_i)$, and of the fluid temperature averaged over the cross-sections of the stream along the heat exchanger, $t_i^* = f(x_i)$. The diagrams

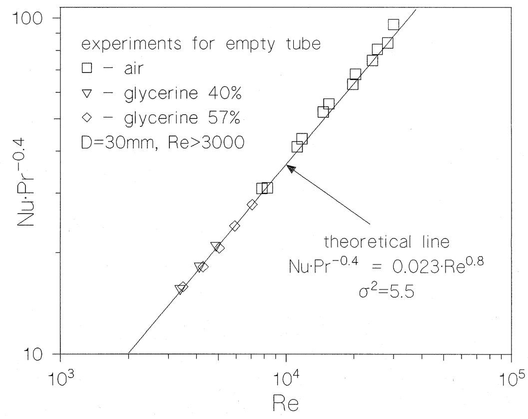



Fig. 5. Results of experiments in ordinary tube.

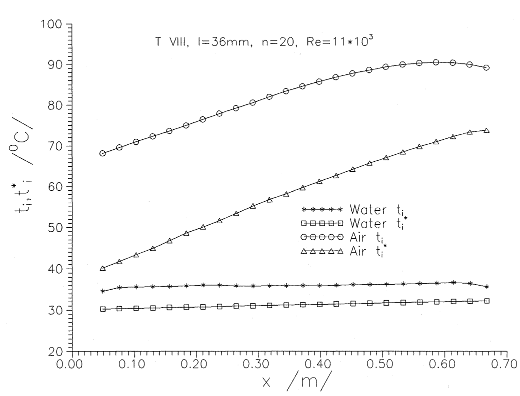



Fig. 6. Distributions of wall temperature, $t(x_i)$, and fluid temperature, $t_i^*(x_i)$, along heat exchanger, $D = 30$ mm.

indicate that for moderate $n$ ($n = 20$), and for relatively large $Re$ ($Re = 11 \times 10^3$), the temperature profiles in the central part of the exchanger are linear both for heating water as well as air. Only along the short inlet and outlet sections are small deviations observed.

Such longitudinal temperature profiles were employed in the evaluation of $Nu_i$ according to the scheme discussed above (Fig. 4). These $Nu_i$ values vary to a degree along the exchanger tube depending on the conditions applied in the individual experiments, as is seen in the diagrams in Fig. 7(a)–(c). The diagrams in Fig. 7(a) indicates that the $Nu_i$ values increase slightly along the exchanger while heating the water stream, but they oscillate about an average value almost constant along the apparatus when the air stream is heated. The diagrams in Fig. 7(b) indicate that the oscillations decrease as the baffles spacing $l$ is reduced. It can also be seen that the reduction of $l$ from 140–36 mm generates an increase in $Nu$, e.g. by about 80%. However, further reduction of $l$, up to 2 mm, does not affect $Nu$ markedly; thus, close packing of the baffles within the heat exchanger tube is not advisable. Figure 7(c) illustrates the effect of the baffle type on the distribution of the local $Nu_i$ values along the exchanger. The investigated baffles differ essentially in their geometry, as can be seen in Fig. 1. Higher values of $Nu$ were observed, both for heating air as well as water streams when baffles T-VIII were applied.

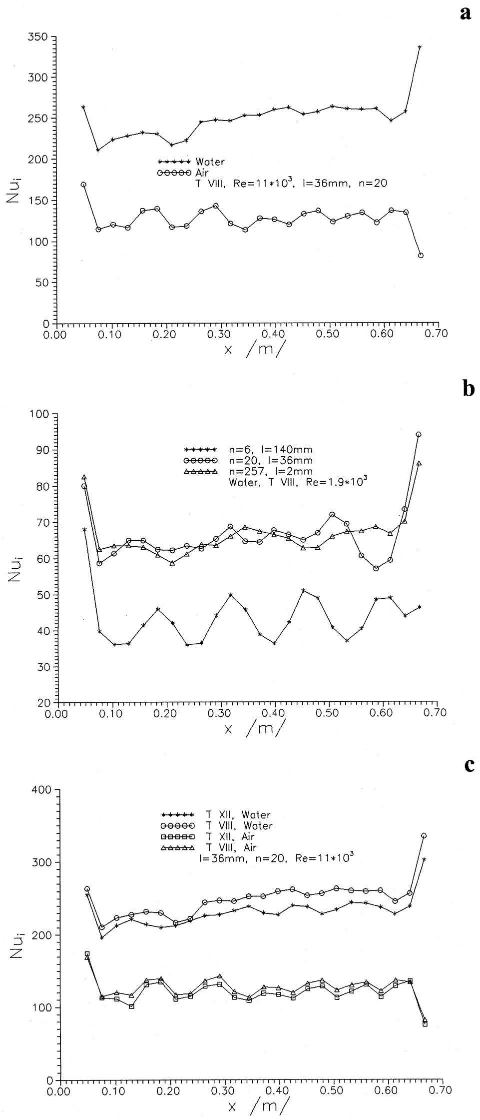



Fig. 7. Distribution of local Nusselt numbers along heat exchanger, $D = 30$ mm (a) effect of fluid nature; (b) effect of baffles spacing; (c) effect of baffle type.

In order to elaborate a general correlation on the basis of the experimental results, it was recognised as advisable to average the $Nu_i$ values over the central part of the exchanger with the length $L_c = 436$ mm in all experiments; thus the end effects were eliminated. This average value of $Nu$ was then employed in equation (3) in order to evaluate accurately the exponents $k$ and $m$, for $Re^+$ and for $Pr$, respectively. The accurate determination of $k$ is now possible due to the experiments performed in a wider range of baffle parameters. The determination of $m$ in our previous work [3] was impossible because only air was used as the heated medium; it was then assumed that $m = 0.4$, as it is for heating fluids in common tubes. In order to verify that assumption, in this work an attempt was made to present results of all experiments, performed with various fluids and with baffle types, as a function of:

$$Nu/APr^{0.4} = (Re^+)^k. \tag{17}$$

Several groups of results correlated in this way are illustrated in Fig. 8(a)–(c). They indicate that $m = 0.399 \pm 0.002 \approx 0.4$. Thus, equation (18) can be formulated with the factor $A$ dependent on the baffle type

$$Nu/Pr^{0.4} = A(Re^+)^{0.7}. \tag{18}$$

The use of equation (18) may be, to some extent, troublesome due to the necessity of experimental determination of $\xi$ or $\xi^+$ involved in $Re^+$, and of the factor $A$. To eliminate such inconvenience, a method has been proposed for evaluating both $\xi^+$ and $A$ on the basis of the baffle parameters.

The method is based on the results of some theoretical considerations concerning fluid flow patterns and on the results of measurements of the pressure drops in the tube furnished with the baffle sets used in the heat transfer experiments.

The results of these measurements are illustrated in Fig. 9(a), (b) showing the dependence of the ratio of the reduced drag coefficient, $\xi^+/\xi_i$, on the ratio of the reduced baffles spacing, $l/d$. It can be seen that the ratio $\xi^+/\xi_i \rightarrow$

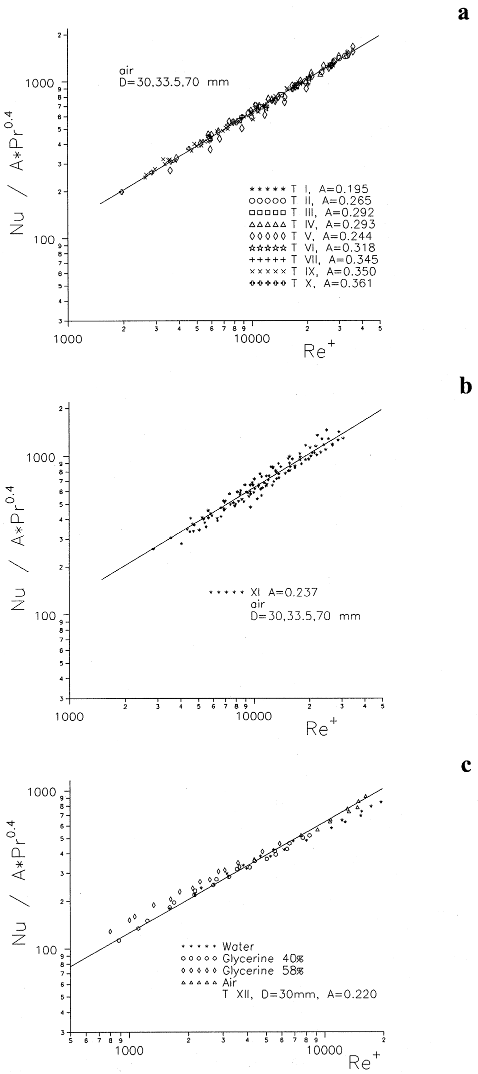



Fig. 8. Effect of modified Reynolds number on heat transfer rate.

1 for a certain value of $l/d$ independent on the fluid nature and on the tube diameter (Fig. 9(a)), but dependent on the baffle type (Fig. 8(b)). This means that when a 'critical' distance between neighbouring baffles in a set is exceeded each baffle influences the fluid stream as an isolated unit.

In order to standardize the experimental results, an effective linear dimension, $d_e$, characterising a baffle has been taken into account. The ratio of this dimension to the opening diameter, $d_e/d$, when $\xi^+/\xi \to 1$ is in the range of $1.8 \leqslant d_e/d \leqslant 4.5$; $d$ denotes real diameter of the identical circular openings and it denotes the hydraulic diameter of the openings in the baffle types T-IX and T-X.

The relations between the ratios $\xi^+/\xi_i$ and $l/d_e$ for several baffle types, different tube diameters and different fluids were analysed and some examples are illustrated in Fig. 10(a), (b). The values of $l = d_e$ at $\xi^+ \approx \xi_i$ for individual baffles were estimated according to the relation

$$d_e/d = [(\rho u^2 L)/(2\Delta p d)][(1+0.707\sqrt{1-\varepsilon}-\varepsilon)/\varepsilon]^2 \tag{19}$$

obtained from equation (20) after expressing $\xi_i$ by equation (8).

$$d_e = l = (\xi_i \rho u^2 L)/2\Delta p. \tag{20}$$

The relations $\xi^+/\xi_i = f(l/d_e)$ for various $Re$, can be correlated by the formula

$$\xi^+/\xi_i = B(l/d_e)^m \tag{21}$$

in which $m = 0$ for $l \geqslant d_e$, $m = 1$ for $l < d_e$, and $B$ depends on $Re$ as follows

$$B = (Re/10^4)^{0.15}. \tag{22}$$

It has been found that $0.67 < B < 1.18$ for $7\times10^2 < Re < 3\times10^4$.

From the analysis of the diagrams in Fig. 10(a), (b) two conclusions follow.

1. For $l \geqslant d_e$, $\xi^+$ depends only on $\varepsilon$ and it can be evaluated from equation (8).
2. For $l < d_e$, $\xi^+$ increases linearly with $l$, thus $\Delta p$ is independent on $n$, as follows from equations (4) and (6).

The reason for this phenomena can be explained on the basis of the radial profiles of the velocity and those of the turbulence measured in the fluid stream between the baffles and illustrated in Fig. 11(a), (b). It can be seen that the profiles measured downstream of the fifth and of the thirteenth baffle in the set, have almost the same shape. This fact indicates that the velocity profiles, corresponding with the minimum dissipation of the kinetic energy of the fluid stream, becomes stable within the baffle set. The deformation of the velocity radial profile in the fluid stream when passing each baffle in the set is insignificant, and, in consequence, the flow resistance is independent on the number of the baffles in the heat exchanger tube. On the basis of the experimental results presented in Fig. 11(a), (b), a simplified mathematical model of fluid flow in a considered system has been formulated. According to the recorded velocity profiles, one may consider the fluid stream within the set of the perforated baffles, each with $n_0$ openings, as a collection of $n_0$ jets and of stagnation regions between them where the velocity profile is approximately flat and the local velocities, $\hat{u}_s$, are considerably smaller than the average velocity referred to the tube cross-section. The jets have the shape of a cone with the diameter of its base, $d_e > d$, and with the height corresponding to the maximal fluid velocity, $\hat{u}_m$, at the centre of the opening. Thus, one may assume that the ratio of the average, $\hat{u}_a$, and of the maxi-

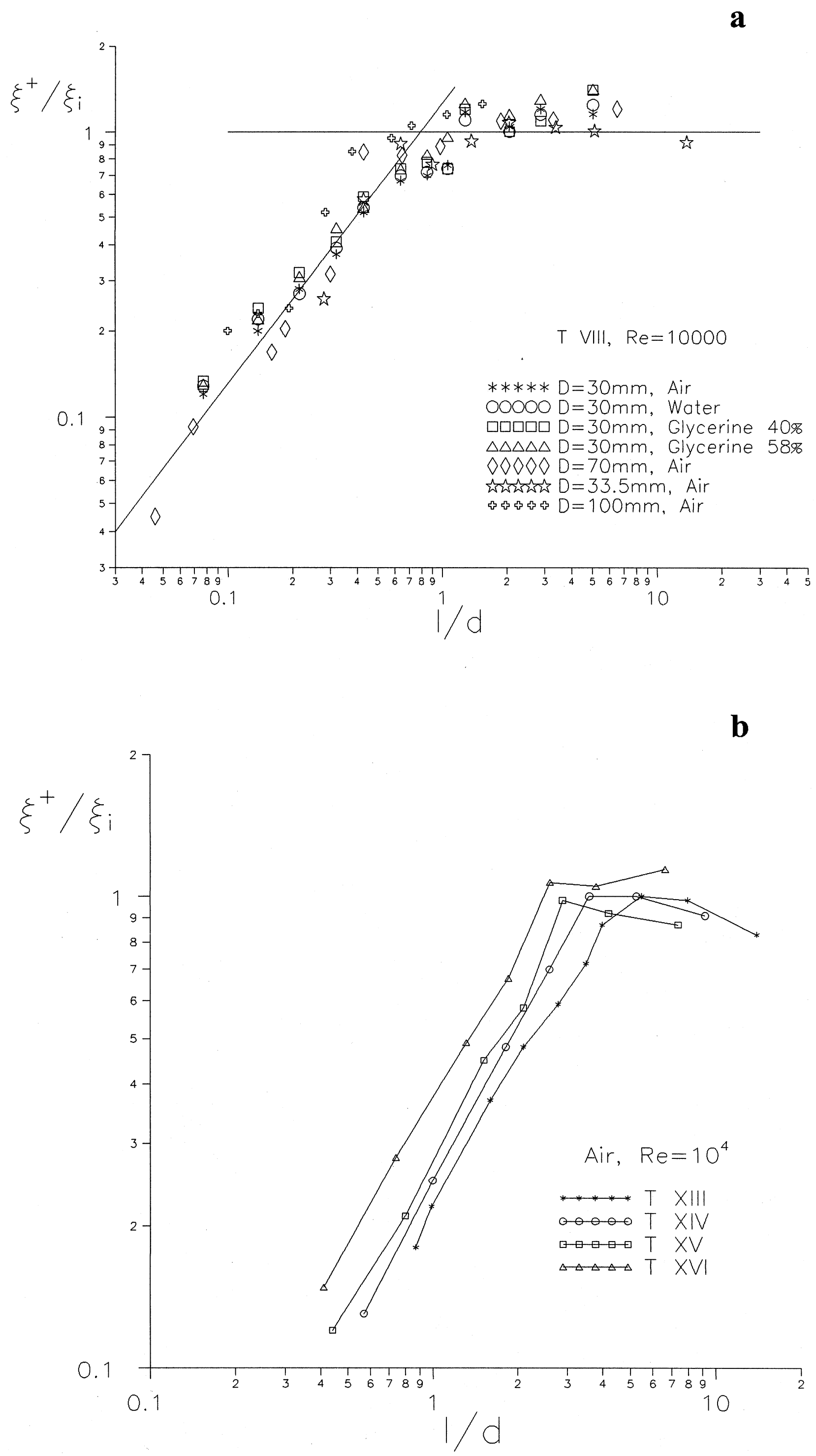



Fig. 9. Relation between reduced drag coefficient, $\xi^+/\xi_i$, and reduced spacing, $l/d$.

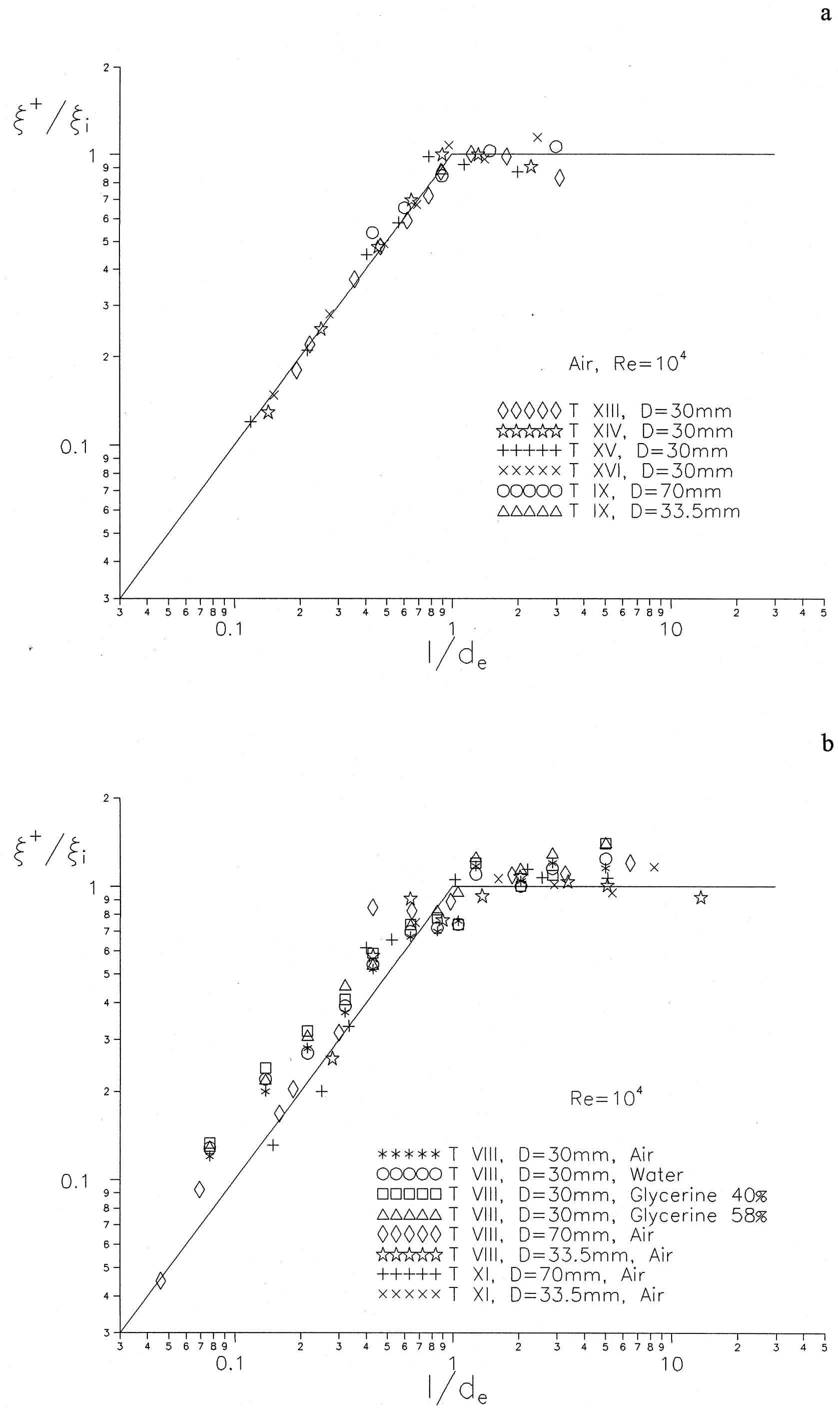
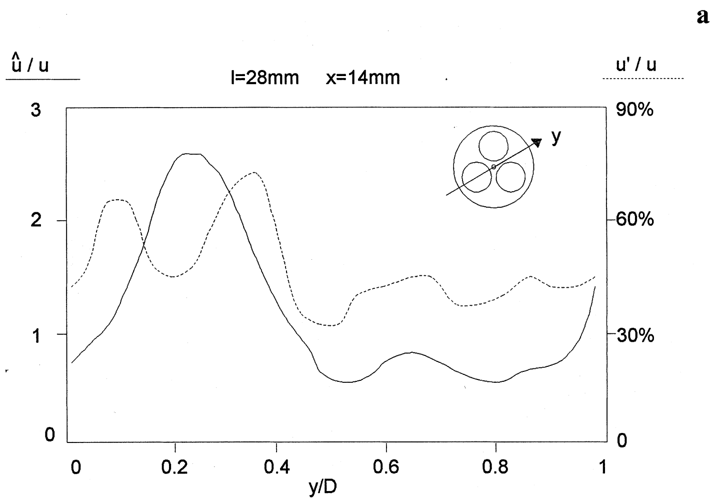



Fig. 10. Relation between reduced drag coefficient, $\xi^+/\xi_i$, and reduced effective spacing, $l/d_e$.

mal, $\hat{u}_m$, velocities within the individual jet can be approximated as:

$$\hat{u}_a/\hat{u}_m = 1/3k. \tag{23}$$

The factor, $k$, dependent on the baffle type, characterises the velocity profile within the jet; e.g., for parabolic profile $\hat{u}_a/\hat{u}_m = 1/2$ and $k = 2/3$, for 'conical' profile $k = 1$, and for a jet with an elongated conical shape $k > 1$.



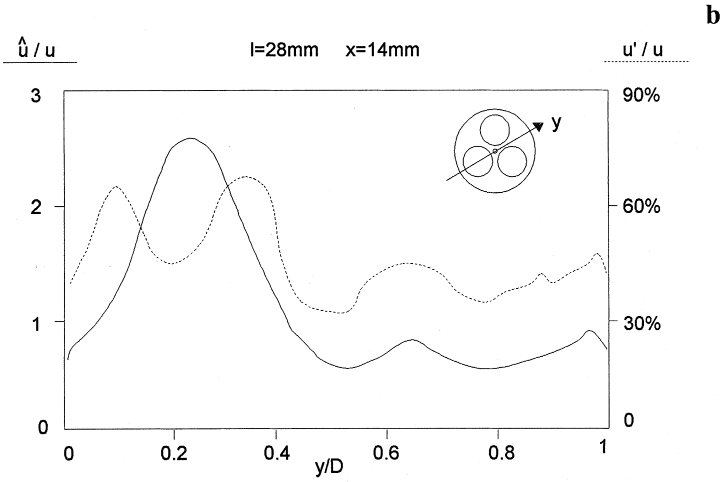




Fig. 11. Radial profiles of velocity, $\hat{u}/u$, and turbulence, $u'/u$, within air stream between baffles (a) downstream the fifth baffle within the set of nine baffles; (b) downstream the thirteenth baffle within the set of 17 baffles. $D = 70$ mm, T-VIII.

The above mentioned mathematical model is based on the mass balance of the isothermal fluid stream in the tube equipped with the baffles and in the empty tube.

$$n_0 d_e^2 \hat{u}_a + (D^2 - n_0 d_e^2)\hat{u}_s = D^2 u. \tag{24}$$

When equation (24) is divided by the term $D^2u$, and some transformations are made, equation (25) is obtained

$$(d_e/D)^2 = (1-\hat{u}_s/u)/[n_0(\hat{u}_a/u - \hat{u}_s/u)]. \tag{25}$$

The number of the openings in the baffle is expressed by the formula:

$$n_0 = (D/d)^2 \varepsilon. \tag{26}$$

Insertion of equations (23) and (26) into equation (25) results in the formulae

$$d_e/d = A(T)\sqrt{3/\varepsilon} \tag{27}$$

$$A(T) = [k(1-\hat{u}_s/u)/(\hat{u}_m/u - 3k\hat{u}_s/u)]^{1/2}. \tag{28}$$

Parameter $A(T)$ characterises the fluid flow field within a set of baffles, and its value depends on the baffle type.

Table 1 presents the exemplary collection of the values

Table 1
Parameters characterising the flow field generated by a perforated baffle

| T | $\hat{u}_s/u$ | $\hat{u}_m/u$ | $k$ | $A(T)$ | $d_e/d$ Equation (19) | $d_e/d$ Equations (27), (28) |
|---|---|---|---|---|---|---|
| VIII | 0.5 | 2.5 | 1 | 0.71 | 1.8 | 1.8 |
| XI | 0.625 | 3 | 4.25/3 | 1.25 | 4.0 | 4.0 |

of the parameters $\hat{u}_s/u$, $\hat{u}_a/u$ and $k$, all estimated on the basis of the fluid velocity profiles measured within the baffle sets T-VIII and T-XI. They were employed in equation (28) for calculating the values of the factor $A(T)$ which, in turn, were used in calculating the ratio $(d_e/d)_{cal}$ according to equation (27). Both the values of $A(T)$ and of $(d_e/d)_{cal}$ are presented in Table 1; for comparison, the values of $(d_e/d)_{exp}$, calculated from equation (19) on the basis of the pressure drops measured, are also presented and perfect agreement is evident.

The analysis of the experimental results has indicated that

$$A(T) = 1 \tag{29}$$

and thus

$$d_e/d = \sqrt{3/\varepsilon} \tag{30}$$

when the opening diameter is sufficiently small, all the openings in the baffle are identical and circular in shape, and their number is sufficiently large;

$$d/D < 1/3, \quad d = \text{const}, \quad \text{and} \quad n_0 > 3. \tag{31}$$

Table 2 presents a comparison of the $(d_e/d)_{cal}$ values estimated according to equation (30) with $(d_e/d)_{exp}$ estimated by means of equation (19) on the basis of the measured pressure drops. The comparison is made for the baffles fulfilling the conditions of equation (31). The agreement is quite satisfactory.

Table 3 comprises the data characteristic for the baffles which do not fulfil the conditions of equation (31). It is seen that the values of $A(T)$ change in the range $0.75 \leqslant A(T) \leqslant 1.50$.

Summing up, the sequence of equations (27), (21) and (5) allows us to predict the values of $d_e$, $\xi^+$ and finally, $\Delta p$, for a given baffle type. The comparison of $\Delta p_t$, estimated according to the method proposed, with those values of $\Delta p_{exp}$, determined experimentally, is shown in Fig. 12; the average deviation is 8%.

Table 2
Comparison of $(d_e/d)_{exp}$ and $(d_e/d)_{cal}$ for baffles for which $A(T) = 1$

| T | $(d_e/d)_{exp}$ Equation (19) | $(d_e/d)_{cal}$ Equation $(30)_1$ | $n_0$ | $d/D$ | $\varepsilon$ |
|---|---|---|---|---|---|
| I | 2.6 | 2.6 | 85 | 0.0714 | 0.44 |
| II | 2.6 | 2.6 | 46 | 0.1 | 0.47 |
| III | 2.6 | 2.3 | 60 | 0.1 | 0.61 |
| IV | 2.6 | 2.7 | 28 | 0.121 | 0.42 |
| V | 2.6 | 2.7 | 20 | 0.143 | 0.42 |
| VI | 2.6 | 2.6 | 11 | 0.2 | 0.45 |
| VII | 2.6 | 2.3 | 15 | 0.2 | 0.61 |
| XII | 2.9 | 3.0 | 36 | 0.096 | 0.40 |

Table 3
Parameters of the baffles for which $0.75 \leqslant A(T) \leqslant 1.5$ and $d_e/d = A(T)\sqrt{(3/\varepsilon)}$

| T | $\varepsilon$ | $d_e/d$ | $\sqrt{(3/\varepsilon)}$ | $A(T)$ | $n_0$ | $d/D$ |
|---|---|---|---|---|---|---|
| VIII | 0.490 | 1.8 | 2.5 | 0.75 | 3 | 0.400 |
| IX | 0.610 | 2.7 | 2.22 | 1.2 | 2 | 0.550 |
| X | 0.610 | 2.6 | 2.2 | 1.2 | 6 | 0.334 |
| XI | 0.300 | 4.0 | 3.2 | 1.3 | 3 | 0.320 |
| XIII | 0.167 | 4.5 | 4.2 | 1.2 | 1 | 0.330 |
| XIV | 0.318 | 4.0 | 3.1 | 1.3 | 1 | 0.510 |
| XV | 0.461 | 3.7 | 2.5 | 1.5 | 1 | 0.633 |
| XVI | 0.555 | 2.7 | 2.32 | 1.2 | 1 | 0.700 |

The analysis of the experimental data concerning the rate of heat transfer in the investigated system indicates that the factor $A$ in equation (18) may be expressed by equation (32) valid for all baffles investigated.

$$A = 0.367(d_e/d)^{0.3}. \tag{32}$$

Thus, equation (18) has been brought to the form

$$Nu = 0.367(d_e/d)^{0.3}Pr^{0.4}(Re^+)^{0.7}. \tag{33}$$

Figure 13 illustrates the fitting of the experimentally determined heat transfer rates with equation (33); the average deviation is $\pm 16\%$ and the maximal deviation is in the range of $\pm 25\%$ which does not exceed the maximal error $\pm 30\%$ estimated on the basis of the error analysis enclosed in the appendix.

The results of our previous investigation has shown that the insert of the perforated disc-baffles T-VIII is the most effective regarding the reduction of the heat transfer surface area in a tube-in-shell type heat exchanger. Such insert has not been described in the literature as yet. Therefore, it is rather difficult to compare its augmenting effects with analogous literature data related exactly to that type of augmenting means.

However, the comparison has been done for the insert of perforated baffles T-XVI and for the augmenting means proposed by Nunner [9] and Webb et al. [10] because those proposals indicate similar, to some extent, geometrical parameters. Their common feature is that the ring-shaped elements disturbing the fluid stream within the tube are uniformly spaced along it. Figure 14 illustrates the heat transfer augmenting effects given by the perforated baffles T-XVI and by the augmenting means proposed by Nunner [9] and Webb et al. [10] in comparison with the heat transfer rate observed in the

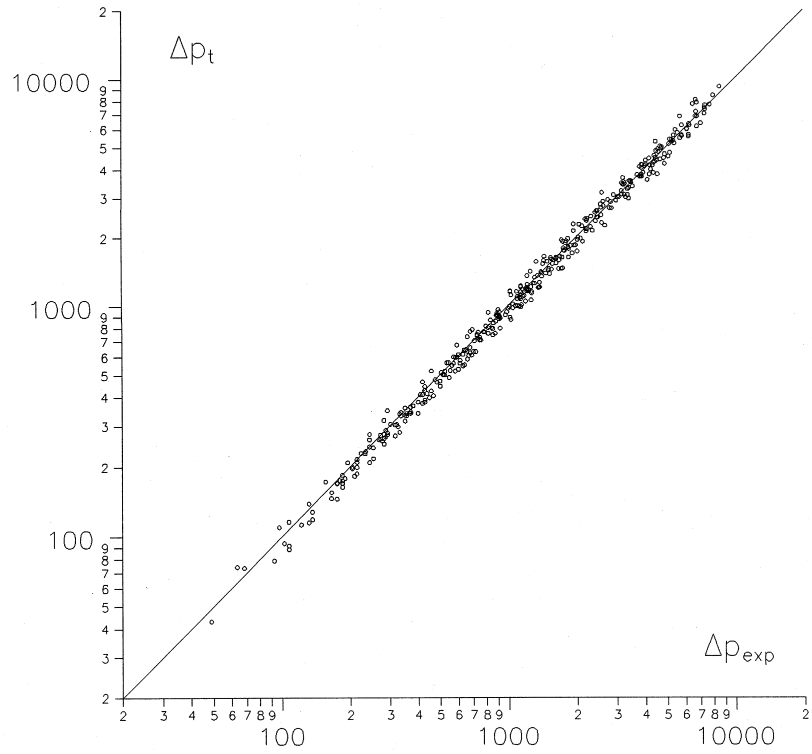



Fig. 12. Comparison of pressure drops calculated, $\Delta p_t$, and experimentally determined, $\Delta p_{exp}$.

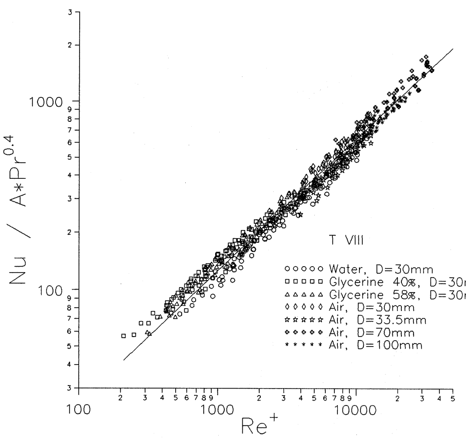



Fig. 13. Comparison of the experimentally determined data with the theoretical line predicted by equation (33).

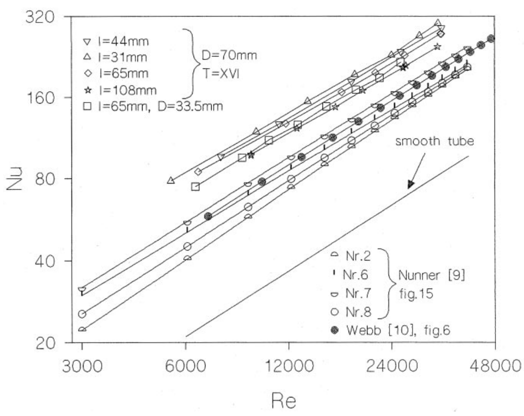



Fig. 14. Comparison of the heat transfer augmentation effects generated in the air stream by the baffles T-XVI and by the augmentation means proposed by Nunner [9] and Webb et al. [10].

ordinary tube. It can be seen that the insert of perforated baffles T-XVI is the most effective.

## 4. Conclusions

(1) The heat transfer during heating a fluid stream in a tube furnished with a set of perforated baffles can be modelled with satisfactory accuracy by means of equation (33) in which $Nu$ and $Pr$ are defined conventionally, but $Re^+$ is based on the shear stress fluid velocity related with the drag coefficient, $\xi^+$, specific for a single baffle in the set.

(2) The values of $\xi^+$, so far available by experiments,

can be predicted by the presented method when the baffle parameters $\varepsilon$, $D$, $d$, $l$ and $d_e/d$ are known. The knowledge of $\xi^+$ value allows us to predict the heat transfer rate as well as the pressure drop in the investigated system.

(3) The parameter, $d_e/d$, characteristic for a given baffle type, can be evaluated according to equation (30) when the openings in the baffle fulfil the conditions of equations (31). For the baffles which do not fulfil such conditions, $d_e/d$ has to be estimated by means of equation (27) with a value of $A(T)$ taken from the range, $0.75 \leqslant A(T) \leqslant 1.50$, as it is seen in Table 3.

(4) The heat transfer rates and the pressure drops predicted according to the proposed method have been verified experimentally in the ranges of $0.7 \leqslant Pr \leqslant 34$ and $7 \times 10^2 \leqslant Re \leqslant 3 \times 10^4$. Satisfactory agreement between the predicted and experimentally determined values is evident.

## Appendix: the error analysis for equation (33)

Equation (33) correlates the dependent variable, $Nu$, with the independent variables $(Re^+)^{0.7}$, $(d_e/d)^{0.3}$ and $Pr^{0.4}$, each of them determined experimentally with certain error. The evaluation of the errors at which these variables have been determined enables us to estimate the correctness of equation (33). The error analysis has been performed for the case with the glycerine aqueous solutions as their physical–chemical properties might be determined with the largest errors in comparison to the case of other fluids used in the heat transfer experiments. The possible errors of the system properties influencing the variables in equation (33) are collected in Table A1.

The maximal error of the individual variables in equation (33) were estimated on the ground of the error analysis applied to the formulae used for their evaluation from the experimental data each laden with an error enclosed in Table A1.

The local Nusselt number, $Nu_i$, was evaluated according to equation (9), hence

$$\frac{\mathrm{d}\,Nu_i}{Nu_i} \cong 17\%. \tag{A1}$$

Table A1
The possible errors of the system properties

| Property | $c_p$ | $\lambda$ | $\mu$ | $\rho$ | $D$ | $G$ | $\Delta p$ | $q_i$ | $t_i$ |
|---|---|---|---|---|---|---|---|---|---|
| | Of correlations | | | | | Of gauges | | | |
| Error | 3% | 3% | 3% | 3% | 0.3% | 5% | 5% | 1% | 0.1°C |

The simplex $(d_e/d)$ was evaluated according to equation (20) in which equation (8) is included, hence

$$\frac{\mathrm{d}(d_e/d)}{(d_e/d)} \cong 15\% \tag{A2}$$

and the maximal error the term $(d_e/d)^{0.3}$ is about 4.5%.

The Prandtl number, $Pr$, was evaluated according to the conventional definition, hence

$$\frac{\mathrm{d}\,Pr}{Pr} \cong 9\% \tag{A3}$$

and the maximal error of the term $Pr^{0.4}$ is about 3.6%.

The modified Reynolds number, $Re^+$, was evaluated according to the formula (A4) resulted from the combination of equations (1), (2) and (4)

$$Re^+ = \frac{D^{3/2}}{2\mu}\sqrt{\frac{\Delta p \rho}{L}} \tag{A4}$$

hence

$$\frac{\mathrm{d}\,Re^+}{Re^+} \cong 7.5\% \tag{A5}$$

and the maximal error of the term $(Re^+)^{0.7}$ is about 5.2%.

Coming to the conclusion, one can say that the maximal error of equation (33) does not exceed $\pm 30\%$. For comparison, the maximal deviation of the experimental results from equation (33) is in the range of $\pm 25\%$ with the standard deviation of $\pm 16\%$, as can be seen in Fig. 13.

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